Equation (16) is now considered for the heat conduction through a thin fin under the assumption that the conductivity-thickness product varies according to

$$k_f t = (k_f t)_b (x/x_b)^s \tag{26}$$

where s is the prescribed fin shape exponent. The substitution of Eqs. (26), (23), and (19) into Eq. (16) yields

$$n(s-n-1)(k_f t/x^2) + \tfrac{1}{2}[(3+4m)n-1]^{1/2}(k\,Rax^{1/2}/x) = 0 \tag{27}$$

The foregoing equation suggests the existence of similarity solutions, namely

$$\left(\frac{kx}{k_f t}\right) Rax^{1/2} = \frac{kx_b}{(k_f t)_b} Rax_b^{1/2} = \frac{2(4-3s)(3-2s)}{[(3+4m)(3-2s)-1]^{1/2}} = Ncc \tag{28a}$$

where

$$n = 3-2s \tag{28b}$$

Therefore, the constraint described by Eq. (24a) is equivalent to

$$s < 4/3 \tag{29}$$

Once the thermal stratification parameter m and the fin shape exponent s for the conductivity-thickness product are given, the so-called "conduction/convection parameter" Ncc may readily be calculated from the last expression on the right-hand side of Eq. (28a). Thus, the unknown value x_b can be determined from Eq. (16a) as

$$x_b = \left[\left(\frac{\alpha\nu}{Kg\beta\Delta T_{w_b}}\right)^{1/2} \frac{(k_f t)_b}{k} Ncc\right]^{2/3} \tag{30}$$

The substitution of Eqs. (22), (23), and (28) into Eq. (19) leads to the following closed-form expression for the isotherms:

$$y/x_b = -\frac{kx_b/(k_f t)_b}{(4-3s)(3-2s)} \left(\frac{x}{x_b}\right)^{2-s} \ell n\left[\left(\frac{T-T_c}{\Delta T_{w_b}}\right)\left(\frac{x}{x_b}\right)^{3-2s} - m\right] \tag{31}$$

In order to illustrate the temperature fields within the porous medium, the isotherms were obtained for the case of an infinitely long copper fin with constant thermal conductivity and thickness. The case was previously treated by Pop et al.[6] for the constant ambient temperature. Computations were carried out assuming $k_f = 376.8$ W/m°C, $t = 0.005$ m, $T_{wb} = 200$°C, $K = 10^{-8}$ m^2, $k = 2.428$ W/m°C, $g = 9.8$ m/s^2, $\beta = 1.8\times10^{-4}$/°C, $\nu = 0.27\times10^{-6}$ m^2/s, $\alpha = 0.63\times10^{-6}$ m^2/s, and the constant ambient temperature $T_e = 15$°C ($m = 0$). The resulting isotherms are plotted in Fig. 4a, which indicates a close agreement between the present solution and the exact solution.[6] As pointed out by Pop et al.,[6] the isotherms near the base curve back as a result of a strong flow acceleration there.

When the base is heated up to a high temperature, the effect of the thermal stratification within the porous medium may no longer be negligible. Thus, illustrative calculations were made with the same base temperature but with the thermal stratification parameter $m = 1$ and the reference temperature $T_c = T_e|_{x\to\infty} = 15$°C. Even for the same fin temperature distributions, the isotherms, as plotted in Fig. 4b, indicate a pattern quite different from that in Fig. 4a. These isotherms do not curve back toward the base but extend horizontally away from the fin surface as a result of the thermal stratification. In this sense, the previous solution for the constant temperature should be regarded as somewhat overidealized.

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Gas Particle Radiator

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Introduction

HIGH specific power (power radiated/radiator mass), small area, and long lifetime are the desirable characteristics of a space radiator. These characteristics will be attained if a low mass and high emissivity ϵ_T that is stable for long periods (7-10 years), can be achieved.

For a tube-type radiator (either a heat pipe or a pumped loop) high emissivity ($\epsilon > 0.8$) is achieved by the use of emissive coatings. Adhesion and emissive stability of these coatings must be obtained for long periods of time if a tube-type radiator is to be a successful space radiator. Generally, the largest mass portion of a tube radiator is the armor that must be used to protect it from meteoroid penetration.

The gas particle radiator (GPR) is a new concept that has the potential for a long lifetime and high emissivity with lower mass than tube radiators. Figure 1 is a conceptual drawing of the GPR. A gas which contains a suspension of fine particles is contained in a sealed volume between the tube radiator and an outer window that are separated by a distance, D. On start-up of the radiator, a temperature gradient will exist across the gas. This temperature gradient will induce a gas flow that will distribute the particles throughout the gas. However, this will have to be demonstrated for a successful GPR. In the

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microgravity of space the particles should remain in suspension. If the window is transparent to the emitted radiation, the gas particle mixture will yield a high, stable emissivity. Past investigations[1] of a soot-containing flame that is similar to the gas particle mixture of the GPR, have yielded large emittances. Obtaining high emissivity without the use of emissive coatings is a major advantage of the GPR.

It would appear that the addition of the gas-particle mixture plus a surrounding window will result in a larger mass for the GPR than a tube radiator. However, the addition of the window "bumper" will provide meteoroid protection for the emitting tube. This window bumper means the combined thickness (window + tube) may be significantly less than the tube-only thickness and still provide the same meteoroid protection.[2,3] Therefore, the GPR may have a lower mass than a tube radiator.

The major problem for the GPR is finding a suitable window material. As will be shown, it is the window transmittance that limits the emissivity. There are suitable choices for the gas and particle materials; however, there is a need to experimentally verify the emissivity predicted in this study.

Suitable window materials and a verified emissivity are the critical issues for the GPR. However, even if these issues can be resolved satisfactorily, other design problems will also have to be solved. It will be necessary to provide a seal between the window and the radiating surface. Also, provision will have to be made for the difference in thermal expansion of the window material and the metallic radiating surface material.

Emissivity of Gas Particle Radiator

A number of studies (summarized in Ref. 1) of the emittance of soot-containing gases have been carried out. These investigations yield large emittances for modest amounts of soot concentration. Seeding with small particles has also been proposed to increase gas absorption of incident radiation[4] or as a means of shielding a surface from incident radiation.[5] The success of the GPR depends on large emittance (or absorption) of the gas-particle mixture. In Ref. 4 large absorption was measured as a function of wavelength, λ, in the range $0.2 \leq \lambda \leq 1$ μm for carbon, aluminum oxide, hafnium carbide, and tungsten particles suspended in water. Particle diameters ranged from 0.02 to 2 μm.

The derivation for the total emissivity, ϵ_T for the GPR is presented in Ref. 6. The most important approximations used in that derivation are that the gas, particles, and radiator surface are at the same temperature, T_p, and the absorption coefficient, k_λ^g, of the gas-particle medium is uniform and is determined by the particle optical properties. It is assumed that the gas is transparent to the emitted radiation. Also, the emissivity of the radiator surface, ϵ_p, is neglected. To obtain the largest emissivity with the minimum fraction of solid particles it is necessary to use particles of small radius,[6] r_d, $(r_d/\lambda < 1)$. In that case the absorption coefficient can be assumed to be

$$k_\lambda^g = (K = \lambda)\varphi, \qquad (r_d/\lambda), < 1 \tag{1}$$

where K is a constant; λ is wavelength; and φ is the volume fraction of particles in the gas-particle medium. Equation (1) is an absorption coefficient that fits the emittance results for soot-containing gases.[1,7] For the emission temperature range of interest for the GPR (300–1200 K), more than 90% of the radiation will be in the range $1 \leq \lambda \leq 75$ μm. Therefore, the particles must satisfy $r_d < 1$ μm. As mentioned previously, it was assumed that the gas and particles are at the same temperature. The conditions for the validity of this approximation are presented in Ref. 6.

An important part of the ϵ_T calculation is the window specular transmission, τ_λ^w. For most window materials τ_λ^w can be approximated as a constant, τ_w, within a prescribed wavelength range, $\lambda_\ell \leq \lambda \leq \lambda_u$, and $\tau_\lambda^w = 0$ outside this wavelength range.[8] Using this window approximation and the assumptions described previously, the following result is obtained for ϵ_T.[6]

$$\epsilon_T = \tau_W F(\lambda_u T_p) \tag{2}$$

For the condition

$$\gamma = (3.6KkT_p/hc)\varphi D > 1 \tag{3}$$

where

$$F(x) = \frac{2\pi hc^2}{\sigma} \int_0^x \frac{du}{u^5 [\exp(hc/ku) - 1]} \tag{4}$$

The quantity $F(x)$ is tabulated in Ref. 1. Other quantities appearing in Eqs. (2) and (3) are the speed of light c, Plank's constant h, Boltzmann's constant k, $(hc/k = 14{,}388$ μm $-K)$, and the Stefan Boltzmann constant $\sigma = (5.67 \times 10^{-8}$ $W/M^2K^4)$. In Fig. 2, ϵ_T/τ_w is shown as a function of $\lambda_u T_p$ for $\gamma > 1$. For large $\lambda_u T_p$, ϵ_T will depend only on the window transmission.

Consider the volume fraction φ necessary to attain large $\gamma (> 1)$. The parameter K is in the range 4–6 $\mu m/\mu$m for soot.[1] Therefore, assume $K \approx 5$ $\mu m/\mu$m. Also, considering the lowest temperature of interest $T_p \approx 300$ K and again using $D \approx 1$ cm the volume fraction necessary to attain $\gamma > 1$ is $\varphi > 2.7 \times 10^{-4}$.

Based on the preceding discussion, $\varphi > 10^{-4}$ should insure the maximum feasible emissivity. However, as Fig. 2 shows, only for $\lambda_u T_p \geq 7 \times 10^3$ μm K will ϵ_T/τ_w be large $(\epsilon_T/\tau_w > 0.8)$. At $T_p \approx 300$ K this means $\lambda_u \geq 20$ μm. There are few materials with good transmission at wavelengths this long. Therefore, the GPR will be more appropriate for higher temperatures.

As discussed, it is the window transmission that determines the GPR emissivity. Therefore, it is critical to have a window with large transmittance in the wavelength range of interest. Since the radiator temperatures of interest are 300–1200 K the majority of emitted radiation will be in the infrared $(\lambda > 1 \mu m)$. Therefore, window materials such as ordinary glass will not be

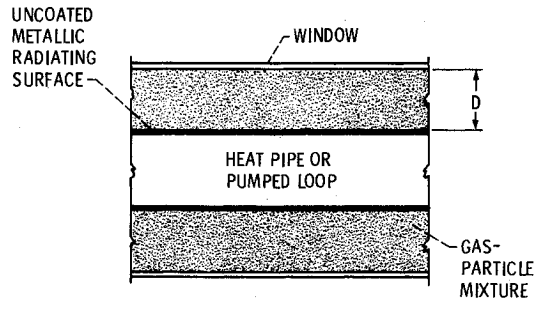

Fig. 1 Gas particle radiator concept.

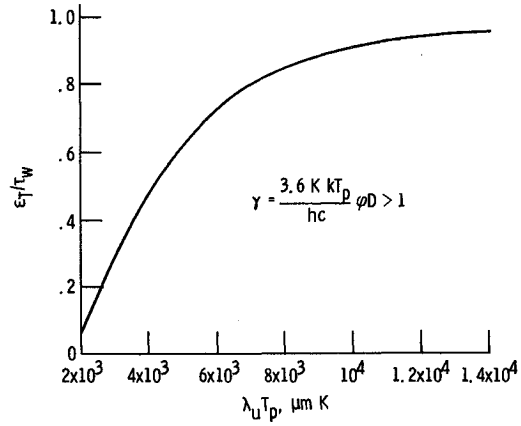

Fig. 2 Total emissivity for small particles ($r_d < 1$ μm, $T_p \leq 1200$ K).

suitable. The alkali halides such as NaCl, NaF, and CsBr have excellent transmittance in the infrared. Also, the heavy-metal flouride glasses[9] are excellent infrared transmitters. Manufacturing large pieces of these materials of the required optical properties may be a problem. However, continuous single crystal fibers of cesium bromide and other alkali halides with excellent transmittance have been manufactured.[10] A list of infrared transmitters and their properties is given in Ref. 6.

Mass Comparison between Gas Particle Radiator and Tube Radiator

Assume that the total radiator mass is proportional to the radiator surface area. Then the ratio of the mass of a GPR, M_T, to the mass of the tube-type radiator, M_0, is the following

$$M_T/M_0 = a_T A_T / a_0 A_0 \tag{5}$$

where a is the proportionality constant.

The area ratio, A_T/A_0, is determined by the power radiated. For a space radiator, redundant area must be included to make up for radiator area lost as a result of meteoroid penetration. As mentioned in the Introduction, the window for the GPR acts as a bumper to protect the radiator surface. Therefore, if the window is penetrated by a meteoroid the radiating gas particle mixture is lost but the radiator surface will remain operable. This remaining surface will have an emissivity ϵ_T' that is less than the GPR emissivity, ϵ_T. For the tube-type radiator all area that is affected by a meteoroid penetration is lost as a radiating surface. The radiator area A_S with missivity ϵ_0 that remains operative at the end of the mission must be sufficient to emit the entire radiator load. In the case of the GPR the radiator load at the end of the mission is shared by two areas. They are the area A_s with emissivity ϵ_T that does not suffer meteoroid penetration, and the area $A_T - A_s$ with emissivity ϵ_T' that has lost the gas particle mixture as a result of meteoroid penetration. If both the GPR and the tube-type radiator are operating with the same load and with the same view factor, the following relation is obtained for the ratio, A_T/A_0:

$$A_T/A_0 = \epsilon_0/\epsilon_T \left[1 + \frac{\epsilon_T'}{\epsilon_T}\left(\frac{1}{r} - 1\right)\right]^{-1} \tag{6}$$

In obtaining Eq. (6) the same redundancy factor r was assumed for the GPR and the tube-type radiator:

$$r = A_s/A_T = A_{s_0}/A_0 \tag{7}$$

In Ref. 6 an expression for a_T/a_0 is developed. This parameter is most sensitive to the thickness of the radiator tube of the GPR, t_p the thickness of the radiator tube of the tube-type radiator, t_0 the window thickness, t_w of the GPR and the thickness of the connecting member between the plate and window of the GPR t_s.

Since the window of the GPR acts as a bumper against meteoroid penetration of the radiating surface the total thickness, $t_p + t_w$ can be significantly less than t_0 and still provide the same meteoroid protection. For high speed meteoroids ($\sim$20 km/s) the bumper fragments and vaporizes the meteoroid dispersing the fragments over a large area so that penetration of the main wall does not occur. Data from Explorer 46[3] using a stainless steel bumper and main wall indicted that $t_p + t_w \approx t_0/6.9$ provided the same meteoroid protection as t_0. In this case, t_w is the bumper thickness. Since meteoroid penetration depth is not sensitive to the target material,[11,12] it is expected that similar behavior will exist for other material. In Ref. 3 the optimum distribution of mass between the bumper and the main wall was calculated to be $t_w/t_p = 0.1$–0.2. Using this result together with $t_p + t_w = t_0/6.9$ the following is obtained:

$$0.12 \le t_p/t_0 \le 0.13 \tag{8}$$

Now consider a comparison between a proposed heat-pipe radiator[12] and a hypothetical GPR. The heat pipe radiator of Ref. 12 uses a flat plate radiating surface of titanium at $T_p = 775$ K with an emissivity, $\epsilon_0 = 0.9$ and a total radiated power of 1.01MW. The radiator wall thickness ($t_0 = 0.60$ mm) and redundancy ($r = 321/360$) was designed to provide a lifetime of seven years with 0.99% probability of survival against meteoroid penetration. The emitting gas particle mixture for the hypothetical GPR consists of small carbon particles and helium. The conditions necessary to insure that $\gamma > 1$ and that negligible temperature difference exists between the particles and gas are developed in Ref. 6.

Window material was chosen to obtain high emissivity. From Fig. 2 it can be seen that $\lambda_u T_p \ge 10^4$ μm K for $\epsilon_T/\tau_w \ge 0.9$. Therefore, for $T_p = 775$ K, $\mu_u \ge 13$ μm. There are several materials[6] that satisfy $\lambda_u \ge 13$ μm and also have a melting point significantly above 775 K. For the hypothetical GPR sodium chloride was chosen as the window material. For NaCl[8] $\lambda_u = 15$ μm and $\tau_w > 0.9$. Using Fig. 2 for $\lambda_u T_p = 1.16 \times 10^4$ μm K, and $\tau_w = 0.9$ results in $\epsilon_T = 0.84$. Thus ϵ_T is slightly less than the assumed emissivity $\epsilon_0 = 0.90$, of the heat pipe radiator. When the area ratio A_T/A_0 [Eq. (6)] is calculated using $\epsilon_T' = 0.3$ and $r = 321/360$ however, it is found that $A_T/A_0 = 1.03$. The increase in area resulting from $\epsilon_0/\epsilon_T > 1$ is nearly compensated for by the redundant area savings $[1 + \epsilon_T'/\epsilon_T\ (1/r - 1)]^{-1}$ term in Eq. (6).

As already mentioned, the thickness of the heat pipe wall is $t_0 = 0.6$ mm. Applying Eq. (15) results in $0.072 < t_p < 0.078$ mm. Although it may be possible to use such a thin wall thickness, $t_p = 0.1$ mm was arbitrarily chosen as a practical limit on t_p. This same limit was also chosen for t_w, so that $t_w = t_p = 1/6\ t_0 = 0.1$ mm. With $t_w + t_p = 0.2$ mm meteoroid protection will be more than required ($t_w + t_p \approx t_0/6.9$). Titanium of thickness $t_s = t_p = 0.1$ mm was also assumed for the connecting member between the plate and window.

Using the materials, wall thicknesses, and dimensions already described, the specific mass ratio a_T/a_0 was calculated[6] and found to be 0.67. From this result plus $A_T/A_0 = 1.03$, Eq. (5) yields $M_T/M_0 = 0.69$. Thus the GPR results in a 31% mass reduction from the comparable heat pipe radiator.

Conclusion

This study was directed at predicting the performance of a new space radiator concept, the gas particle radiator (GPR). The GPR uses a gas containing submicron particles as the radiating media. The gas particle mixture is contained between the radiator's emitting surface and a transparent window. The GPR has two major advantages over conventional heat-pipe or pumped loop radiators. First, high emissivity is achieved without the use of emissive coatings. Second, the GPR potentially has a lower mass.

For a modest volume fraction ($\varphi > 10^{-4}$) of submicron particles and gas thickness ($D \approx 1$ cm) it was found that the emissivity was determined by the window transmittance. Thus the window becomes a critical element in the GPR concept. The window must have high transmittance in the infrared and be structurally strong enough to contain the gas particle mixture. In addition, the window acts as a "bumper" to provide meteoroid protection for the radiator wall. This results in lower mass for the FPR.

The GPR was compared to a proposed titanium wall, potassium heat pipe radiator. For both radiators operating at a power level of 1.01 M_W at 775 K it was found that the GPR mass was 31% lower than the heat pipe radiator.

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